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A Technique for Estimating the Conditional Probability of Detecting a Non-Gaming Drug User

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In order to estimate the effectiveness of random uninalysis drug testing strategies, it is necessary to estimate the conditional probability of detecting drug users; that is, the probability that the user will test positive if selected for drug testing. This report presents a series of methodologies for estimating the conditional probability of detecting non-gaming drug users under a wide variety of drug usage and wear-off scenarios. The most general scenario assumes that the probability of detection depends upon the pattern of drug use prior to testing. For non-gaming users, it was observed that the probability of detection and the expected number of months until detection are proportional to the command monthly test rate.

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Foreword

This report was prepared as part of the Statistical Methods for Drug Testing project (Program Element 0305889N, Work Unit 0305889N.R2143DR001), sponsored by the Chief of Naval Personnel (PERS-63). The objective of the project is to develop a unified set of statistical methodologies for the analysis of drug testing programs and data. The work described here was performed during FYs94-95.

This report extends the methodology developed in the report *Markov Chains for Random Urinalysis III: Daily Model and Drug Kinetics* to more complex patterns of drug usage.

We thank Dr. David Blank of PERS-63 for his constructive input into the problem definition and development of this manuscript.

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Summary

Background

The U.S. Navy has maintained a zero tolerance drug policy since 1981 and has pursued an aggressive drug abuse, detection, and deterrence program. A major component of this effort has been a large-scale urinalysis testing program. All officer and enlisted personnel are subject to random urinalysis testing on a continuous basis.

Previous research (Thompson & Boyle, 1994; Thompson, Boyle, & Hentschel, 1993; Boyle, Hentschel, & Thompson, 1993; Evanovich, 1985) have developed Markov models for analyzing random urinalysis testing strategies. In particular, Thompson and Boyle (1994) present daily Markov models that include drug excretion rate kinetics.

In order to estimate the effectiveness of random urinalysis drug testing strategies, it is necessary to estimate the conditional probability of detecting drug users; that is, the probability that the user will test positive if selected for testing. An important segment of drug users consists of non-gaming individuals; users who choose their specific days of drug usage without regard to urinalysis drug testing strategy.

Objective

The objective of this work is to develop a series of methodologies for estimating the conditional probability of detecting non-gaming drug users under a wide variety of drug usage and wear-off scenarios.

Approach

Three methodologies are developed based upon increasingly less restrictive assumptions concerning drug usage and wear-off. The simplest case assumes that drugs remain in the system during a specific wear-off period and the individual tests positive with certainty whenever a drug test is administered during this time frame. A less stringent case is also analyzed, which assumes that drugs remain in the system for a finite wear-off period but the probability of detection is based upon time since most recent drug use. The third, and most general scenario, assumes that the probability of detection depends upon the pattern of use prior to testing. These scenarios are analyzed using variations of the hypergeometric probability distribution.

Results

Analysis of the three methodologies revealed that it is possible to combine the least restrictive (and most complex) approach with an application of the geometric probability distribution to build models to estimate both the probability of detection during a month and the average duration (in months) until detection. For non-gaming drug users, it was observed that the probability of detection and the expected duration until detection are approximately proportional to the command monthly test rate. Thus, a command that tests an average of 20% of its personnel monthly can expect to detect approximately double the number of its non-gaming users within a testing month

(and detect them in approximately half as many months) as a command that tests an average of 10% of its personnel monthly.

Conclusions and Recommendations

The methodologies presented in this report provide a useful approach for analyzing the probability of detecting non-gaming drug users. The analyses show that command monthly test rates have a dramatic impact on the expected number of months until detection of a non-gaming drug user.

It is recommended that this approach be used as part of the Navy's Drug Policy Analysis System (DPAS). Additionally, it is recommended that models be developed for estimating the cost to the Navy of an undetected drug user as a function of the length of time undetected. These costs in conjunction with the methodologies developed in this report will be an essential input to the formulation of models to assist in the development of an optimal Navy drug testing program.

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Introduction

The U.S. Navy has maintained a zero tolerance drug policy since 1981 and has pursued an aggressive drug abuse, detection, and deterrence program. A major component of this effort has been a large-scale urinalysis testing program. All officer and enlisted personnel are subject to random urinalysis testing on a continuing basis. Current Policy (Chief of Naval Operations [CNO], 1994) requires Navy commands to test 10 to 30% of their members every month. The testing program is intended to deter and detect drug abuse (as well as provide data on the prevalence of drug abuse), and has been successful in reducing the proportion of individuals testing positive for drugs from 7% in 1983 to 1% in 1991.

Previous reports (Thompson & Boyle, 1994; Thompson, Boyle, & Hentschel, 1993; Boyle, Hentschel, & Thompson 1993, Evanovich 1985) have developed Markov models for analyzing random urinalysis strategies. In particular, Thompson and Boyle (1994) present daily Markov models that include drug excretion rate kinetics. These models allow for a fixed length cycle (e.g., weekly, monthly) and treat initial drug dose as a random variable. The models considered drug detection time in urine and noted that they vary by specific drug, dose, physical condition, fluid intake, and frequency of ingestion (Ambre, Ruo, Nelson, & Belknap, 1988; Beckett & Rowland, 1965; Hamilton, Wallace, Shimek, Land, Harris, & Christenson, 1977; Johansson, Gillespie, & Halldin, 1990; Johansson & Halldin, 1989; Cook, Jeffcoat, Sadler, Hill, Voyksner, Pugh, White, & Perez-Reyes,1992). Drug user gaming strategies were developed and corresponding survival probabilities were estimated.

In order to estimate the effectiveness of random urinalysis drug testing strategies, it is necessary to estimate the conditional probability of detecting drug users; that is, the probability that a user will test positive if selected for testing. An important segment of drug users consists of non-gaming individuals; users who choose their specific days of drug usage without regard to urinalysis drug testing strategy. This report extends previous research by developing methodologies for estimating the conditional probability of detecting non-gaming drug users under a wide variety of drug usage and wear-off scenarios. Three methodologies were developed based upon increasingly less restrictive assumptions concerning drug use and wear-off. The simplest case assumes that drugs remain in the system over some time frame and the individual tests positive with certainty whenever a drug test is administered during this wear-off period. A less stringent case assumes that drugs remain in the system for a finite wear-off period but the probability of testing positive is based upon time since most recent drug use. The third, and most general scenario, assumes that the probability of testing positive depends upon the pattern of drug use prior to testing. These conditional probabilities can then be input into the Navy's Drug Policy Analysis System (DPAS) to determine the probability of detecting a drug user and other relevant outcome measures under alternative urinalysis testing scenarios. Under certain assumptions, these conditional probabilities can also be used to analyze the likelihood of detecting users under drug gaming scenarios.

Methodology

Let M represent the number of days in a month; let w represent the number of days of drug wear-off (w is assumed to be no greater than M-1 days); let α represent a specific day of the month and let r represent the number of days per month that the non-gaming user uses drugs.

Scenario 1: Drug wear-off is unitary; that is, the user tests positive with probability equal to 1 whenever testing is conducted within w days after drug consumption. Let α represent the conditional probability of testing positive on day a and let $\alpha' = 1 - \alpha$ represent the probability of testing negative on day a. Then, under the assumption that the user is non-gaming, that is, all days are equally likely to be chosen as drug usage days:

$$\alpha = 1 - \alpha' = 1 - \frac{\binom{M-w}{r}}{\binom{M}{r}} . \tag{1}$$

Equation (1) is based upon the hypergeometric distribution (Feller, 1957). In order for a non-gaming individual to test negative on day a, he must use drugs on r days that will not lead to a positive test on day a; that is, he must not use drugs on days $a - 1, a - 2, \ldots a - w$. Therefore, he must choose his r days of drug usage without replacement from the remaining $M - w$ days in the month. We assume that drugs are used after the time of day when drug tests are conducted; therefore, drug use on a test day will not affect the individual's probability of detection on that day. Since α only takes on the values 1 and 0 (i.e., is binary), it follows that the expected value of α, $E(\alpha)$, is given by (1). A four day wear-off scenario is diagrammed in Figure 1.

Day	a-5	a-4	a-3	a-2	a-1	a
Use Drugs	Can	Can't	Can't	Can't	Can't	Can

Figure 1. Allowable patterns of drug usage to avoid detection on day a with four day wear-off.

Example 1: Consider a 'seven' day time frame where a non-gaming user plans to use drugs twice during the period. Suppose the drug has a four day wear-off. Under this scenario,

$$E(\alpha) = 1 - \frac{\binom{M-w}{r}}{\binom{M}{r}} = 1 - \frac{\binom{7-4}{2}}{\binom{7}{2}} = 1 - \frac{3}{21} = .857. \tag{2}$$

Example 2: Consider a 'seven' day time frame where a non-gaming user plans to use drugs four times during the period. Suppose the drug has a four day wear-off. Under this scenario,

$$E(\alpha) = 1 - \frac{\binom{M-w}{r}}{\binom{M}{r}} = 1 - \frac{\binom{7-4}{4}}{\binom{7}{4}} = 1 - \frac{0}{35} = 1. \tag{3}$$

Scenario 2: Drug wear-off is non-unitary; that is, $\alpha = \alpha_i$ $(0 \le \alpha_i \le 1)$, $(1 \le i \le w)$ i days after drug usage; $\alpha = 0$ elsewhere. We assume that probabilities are not cumulative. Let p_i denote the probability that the individual uses drugs exactly i days before a and does not use drugs on all $i - 1$ days between a- i and a, not inclusive. Then, under the assumption that the user is non-gaming:

$$p_i = \frac{\binom{M-i}{r-1}}{\binom{M}{r}} . \tag{4}$$

Equation (4) follows from the reasoning that the user must use drugs on day a - i, but cannot use drugs on all days, d, such that $a - i < d < a$. Therefore, the user must choose his remaining r - 1 days of drug usage from the remaining M - i days in the period. A four day wear-off scenario such that $\alpha = \alpha_2$; that is, the last day of drug use was exactly two days before testing is diagrammed in Figure 2. Note that drug use prior to two days before testing is not relevant under this scenario. It follows that:

$$E(\alpha) = \sum_{i=1}^{w} \alpha_i p_i = \sum_{i=1}^{w} \alpha_i \frac{\binom{M-i}{r-1}}{\binom{M}{r}} . \tag{5}$$

Day	a-5	a-4	a-3	a-2	a-1	a
Use Drugs	Can	Can	Can	Must	Can't	Can

Figure 2. Allowable patterns of drug usage such that $\alpha = \alpha_2$

Example 3: Consider a 'seven' day time frame where a non-gaming user plans to use drugs twice during the period. Suppose the drug has a four day wear-off with the following probabilities of detection: .9, .7, .5, .3. Under this scenario,

$$E(\alpha) = \sum_{i=1}^{w} \alpha_i p_i = \sum_{i=1}^{w} a_i \frac{\binom{M-i}{r-1}}{\binom{M}{r}} = .9\frac{\binom{6}{1}}{\binom{7}{2}} + .7\frac{\binom{5}{1}}{\binom{7}{2}} + .5\frac{\binom{4}{1}}{\binom{7}{2}} + .3\frac{\binom{3}{1}}{\binom{7}{2}} = .562. \quad (6)$$

Example 4: Consider a 'seven' day time frame where a non-gaming user plans to use drugs five times during the period. Suppose the drug has a four day wear-off with the following probabilities of detection: .9, .7, .5, .3. Under this scenario,

$$E(\alpha) = \sum_{i=1}^{w} \alpha_i p_i = \sum_{i=1}^{w} a_i \frac{\binom{M-i}{r-1}}{\binom{M}{r}} = .9\frac{\binom{6}{4}}{\binom{7}{5}} + .7\frac{\binom{5}{4}}{\binom{7}{5}} + .5\frac{\binom{4}{4}}{\binom{7}{5}} + .3\frac{\binom{3}{4}}{\binom{7}{5}} = .833. \quad (7)$$

Scenario 3: Probabilities may be cumulative; that is, $\alpha = \alpha_u$ is dependent upon the specific pattern of drug usage. Let $u \in U = (u_{a-w}, u_{a-w+1}, \ldots, u_{a-i}, \ldots, u_{a-1})$ represent a vector of binary-valued elements, which represent use or non-use of rugs on days $a - w, a - w + 1, \ldots a - i, \ldots a - 1$, respectively. Let $s(u)$ represent the sum of the non-zero elements in u; that is, the number of days the individual used drugs during the w days preceding a.[1] Let $p_a(u)$ represent the probability that $U = u$, a specific pattern of drug usage during the w days preceding a. Then, under the assumption that the user is non-gaming:

$$p_a(u) = \frac{\binom{M-w}{r-s(u)}}{\binom{M}{r}} . \quad (8)$$

A specific pattern of $s(u)$ days of usage can only occur when $r - s(u)$ days of drug usage are chosen from the remaining $M - w$ days in the period. Equation (8) follows since the user is non-gaming, that is, all days are equally likely to be chosen. From equation (8), it follows that:

$$E(\alpha) = \sum_{s(u)=1}^{s(u)=w} \sum_{u \in s(u)} \alpha u \frac{\binom{M-w}{r-s(u)}}{\binom{M}{r}} . \quad (9)$$

Example 5: Consider a 'seven' day time frame where a non-gaming user plans to use drugs twice during the period. Suppose the drug has a two day wear-off with the following probabilities of detection: .5, .2. Furthermore, assume that when drugs are used on consecutive days, the probability of detection rises to .6. Here,

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[1]For example, (0,0,1,1) indicates drug usage on both the first and second day, but not the third and fourth day proceeding day a. In this case $s(u) = 2$; that is, drugs were used on two days.

$$E(\alpha) = \sum_{s(u)=1}^{s(u)=w} \sum_{u \in s(u)} \alpha_u \frac{\binom{M-w}{r-s(u)}}{\binom{M}{r}} = (.5+.2)\frac{\binom{5}{1}}{\binom{7}{2}} + .6\frac{\binom{5}{0}}{\binom{7}{2}} = .195. \quad (10)$$

Application

The probabilities of testing positive for cocaine abuse can be determined from the distribution of the initial dose, x_o. From Thompson and Boyle (1994), page. 7, the probability of testing positive at any time t (in hours) after ingestion is:

$$= P[Y_{BZ(t)} \geq 150 ng/ml] \quad (11)$$

where $Y_{BZ(t)}$ represents the urine concentration for the cocaine metabolite benzoylecgonine (*BZ*). Equation 11 assumes a detectable threshold of 150 *ng/ml*. Thompson and Boyle (1994) obtained concentrations of cocaine and benzolylecgonine for 1,076 urinalysis tests from the San Diego Navy Drug Screening Laboratory and estimated corresponding values of x_o. From these values, $\alpha(t)$ can be estimated as:

$$\hat{\alpha}(t) = \frac{\#\text{ of } x_0 \text{ values with } Y_{BZ(t)} \geq 150}{1076} \quad . \quad (12)$$

From Thompson and Boyle (1994) page 5, equation (10):

$$Y_{BZ}(t) = 883.1863 x_0 [e^{-.0923t} - e^{-.464t}] \, . \quad (13)$$

From (13), assuming drug ingestion at 10:00 p.m. and testing at 10:00 a.m., Thompson and Boyle (1994) obtained a vector of daily wear-off constants that appear in Table 1:

Table 1

Probabilities of Testing Positive for Cocaine Use 1 to 7 Days After Ingestion

Day	t (hours)	a(t)
1	12	.9376
2	36	.6508
3	60	.1229
4	84	0
5	108	0
6	132	0
7	156	0

We assume that the conditional probabilities of drug detection, $\alpha(t)$ are cumulative in the following way. If drugs are used on two consecutive days before a testing day (assuming a two day

wear off), then the conditional probability of detection, α_u, is $\alpha_1 + \alpha_2(1 - \alpha_1) = 1 - (1 - \alpha_1)(1 - \alpha_2)$. That is, we add to α_1 the additional probability of detection if drug use on the day before testing goes undetected but drug use two days before is detected. The equation for any sequence of drug usage, U, as defined in Scenario 3 can be written as:

$$\alpha_u = 1 - \prod_i (1 - \alpha_i) \tag{14}$$

where i represents the non-zero elements of U.

Table 2 presents the estimates of α_u derived from (14) and the data in Table 1.

Table 2

Probabilities of Testing Positive for Cocaine Based on Pattern of Use

Pattern	α_u
(0, 0, 0)	0
(0, 0, 1)	.9376
(0, 1, 0)	.6503
(1, 0, 0)	.1229
(0, 1, 1)	.9782
(1, 0, 1)	.9453
(1, 1, 0)	.6933
(1, 1, 1)	.9809

Consider the situation of a non-gaming individual who uses drugs three times during a 28-day month. From (9),

$$E(\alpha) = (.9376 + .6503 + .1229)\frac{\binom{25}{2}}{\binom{28}{3}} + (.9782 + .9453 + .6933)\frac{\binom{25}{1}}{\binom{28}{3}} + .9809\,\frac{\binom{25}{0}}{\binom{28}{3}} = .1769.$$

Let m_r represent the monthly drug testing rate at a command; that is, the proportion of the command expected to be tested during a (28-day) month. Let k represent the number of test days per month. We assume that the test rate on each test day is the same; that is, $\frac{m_r}{k}$. Since the user is non-gaming, it follows that the probability of detection during a month, $P(det)$, is given by:

$$P(det) = 1 - \left(1 - \frac{m_r}{k} E(\alpha)\right)^k. \tag{15}$$

Equation (15) is derived as follows. The probability of detection on a test day is $\frac{m_r}{k}E(\alpha)$; that is, the individual must be selected for testing and test positive. Therefore, the probability of avoiding detection on all k test days is $\left(1-\frac{m_r}{k}E(\alpha)\right)^k$. If testing is conducted only once per month, then equation (15) reduces to:

$$P(det) = m_r E(\alpha). \tag{16}$$

In this case, the probability of detection is directly proportional to the monthly test rate. Although (16) is not true in general, for small values of $\frac{m_r}{k}$, $\left(1-\frac{m_r}{k}E(\alpha)\right)^k \cong 1 - m_r E(\alpha)$. Therefore, for small values of $\frac{m_r}{k}$,

$$P(det) \cong m_r E(\alpha). \tag{17}$$

The expected (or average) number of months until detection, E(*det*) is given by:

$$E(det) = \frac{1-P(det)}{P(det)} + .5. \tag{18}$$

Equation (18) is derived from the geometric distribution (Feller, 1957), since this problem is equivalent to determination of the expected number of independent Bernoulli trials until a success is achieved. The mean of the geometric distribution has been incremented by .5 under the assumption that detection occurs, on average, at the mid-point of the period. In this example, a trial is a month and a success is detection during the month. Table 3 presents *P*(*det*) and *E*(*det*) for commands (with relatively small daily test rates) using monthly test rates of .05, .10, .15, .20, and .25.

Table 3

Probability of Detection and Average Duration (in Months) Until Detection for Selected Monthly Test Rates

	Monthly Test Rate				
	.05	.10	.15	.20	.25
P(det)	.00885	.01769	.02654	.03538	.04423
E(det)	112.56	56.03	37.19	27.56	22.11

Table 3 shows that *P*(*det*) increases linearly as a function of the monthly test rate. A low monthly test rate of .05 results in a .00885 probability of detection of a three day per month non-gaming drug user while a test rate of .20 increases the likelihood of detection to .03528, a four-fold

increase. Similarly, *E(det)* decreases from 113 months when a .05 monthly test rate is used to approximately 28 months when a .20 rate is used, a reduction in time of 75%. The relationship between monthly test rate and E(*det*) is graphically depicted in Figure 3.

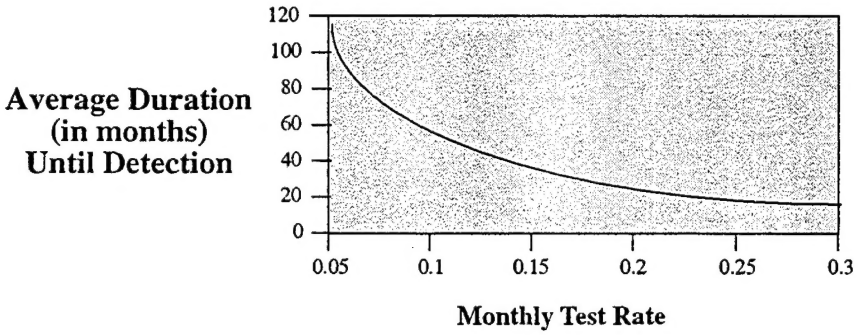

Figure 3. Average duration (in months) until detection for selected monthly test rates.

Conclusions and Recommendations

Three models for estimating the conditional probability of detecting a non-gaming drug user were developed. The models were based upon alternative assumptions about the probability of drug detection during the drug's wear off period. These scenarios included detection with certainty during the period, detection with decreasing probability during the period, and detection based upon pattern of use during the period. Model scenarios incorporated drug kinetics and a cumulative property of drug usage based upon conditional probabilities of detection. For a given command monthly test rate, these models can be used to estimate both the probability of detecting a non-gaming drug user and the average number of months until detection.

The analyses in this report have shown that command monthly test rates have a dramatic impact upon the probability of detection during a month and the expected number of months until detection. In general, an increase in the monthly test rate exerts a proportionate increase in the probability of detection and a corresponding decrease in the average number of months until detection. Thus, a doubling of the monthly test rate was shown to cut in half the average time until detection.

The model development technologies developed in this report can also be incorporated into methodologies for analyzing the gaming drug user.

It is recommended that this approach be used as a part of the Navy's Drug Policy Analysis System (DPAS). Additionally, it is recommended that models be developed for estimating the daily and monthly cost of a drug user remaining undetected. These costs, as a function of time until detection, will be an essential input to the formulation of models to assist in the development of an optimal Navy drug testing program.

References

Ambre, J., Ruo, T. I., Nelson, J., & Belknap, S. (1988). Urinary excretion of cocaine, benzoylecgonine, and ecgonine methyl ester in humans. *Journal of Analytic Toxicology, 12*, 301-306.

Beckett, A. H., & Rowland, M. (1965). Urinary excretion kinetics of amphetamine in man. *Journal of Pharmacy and Pharmacology, 17*, 628-639.

Boyle, J. P., Hentschel, D. J., & Thompson, T. J. (1993). *Markov chains for random urinalysis II: Age-test model with absorbing state* (NPRDC-TN-93-6). San Diego: Navy Personnel Research and Development Center.

Cook, C. E., Jeffcoat, A. R., Sadler, B. M., Hill, J. M., Voyksner, R. D., Pugh, D. E., White, W. R., & Perez-Reyes, M. (1992). Pharmacokinetics of oral methamphetamine and effects of repeated daily dosing in humans. *Drug Metabolism and Disposition, 20(6)*, 856-862.

Evanovich, P. (August 1985). *A model for drug testing* (CRM 85-33). Alexandria, VA: Center for Naval Analyses.

Feller, W. (1957). *An introduction to probability theory and its applications, volume I.* New York: John Wiley & Sons, Inc.

Hamilton, H. E., Wallace, J. E., Shimek, E. L., Land, P., Harris, S. C., & Christenson, J. G. (1977). Cocaine and benzoylecgonine excretion in humans. *Journal of Forensic Sciences, 22(4)*, 697-707.

Johansson, E., Gillespie, H. K., & Halldin, M. M. (1990). Human urinary excretion profile after smoking and oral administration of [^{14}C]Δ^1-tetrahydrocannabinol. *Journal of Analytical Toxicology, 14*, 176-180.

Johansson, E., & Halldin, M. M. (1989). Urinary excretion half-life of Δ^1-tetrahydrocannabinol-7-oic acid in heavy marijuana users after smoking. *Journal of Analytical Toxicology, 13,* 218-223.

Thompson, T. J., Boyle, J. P., & Hentschel, D. J. (1993). *Markov chains for random urinalysis I: Age-test model* (NPRDC-TN-93-5). San Diego: Navy Personnel Research and Development Center.

Thompson, T. J., & Boyle, J. P. (1994). *Markov chains for random urinalysis III: Daily model and drug kinetics.* (NPRDC-TN-94-12). San Diego: Navy Personnel Research and Development Center.

Thompson, T. J., & Boyle, J. P. (1994). Models for detection of drug use in the U.S. Navy. *Socioeconomic Planning Sciences., 28(1),* 9-18.

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